JMRF, Vol. I, No. 1, 50-54, 2023
web: https://publication.morancollege.com/journals/JMRF/2023/article10.pdf

Topological Magnetoelectric Effects in a Semiconductor Quantum Wells via method of image Dyons

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Abstract
This study investigates the influence of nanostructure shape and size on magnetic fields induced by point charges near topological-insulator (TI) and ordinary-insulator (OI) interfaces. Employing the method of image dyons, this work explores the complex magnetic field behaviours that arise when charges are positioned within semiconductor quantum wells (QWs) encased by TIs, focusing on both centred and off-centred charge configurations. Notably, our results reveal the precise engineering of magnetic field configurations via the topological magnetoelectric effect (TME) and the profound impact of nanostructure geometry. Moreover, our study's robustness is demonstrated through the close alignment of results with prior research. The findings open exciting avenues for innovative materials and device development in condensed matter physics and technology, showcasing the method of image dyons as a promising and reliable analytical tool for understanding complex magnetic field interactions in nanostructures near TI-OI interfaces.

Keywords
Topological Insulators; Topological Magnetoelectric Effect (TME); Semiconductor Quantum wells

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1. Introduction

The magnetoelectric effect, where the presence of static electric fields can lead to the induction of magnetic polarization, and conversely, magnetic fields can generate electric polarization, has garnered significant attention in the realm of quantum field theory [1]. This intriguing phenomenon has been both anticipated and observed in a multitude of condensed matter systems, with notable prominence in the context of topological insulators (TIs) [2]. TIs possess a unique attribute, quantized magnetoelectric polarizability, setting them apart from ordinary insulators (OIs) where this property is notably absent [3]. Recent experimental validations demonstrating the existence of the topological magnetoelectric effect (TME) in TI materials have ignited considerable interest in potential technological applications [4].

A remarkable manifestation of the TME takes center stage at the interface between TIs and OIs, particularly when localized magnetization disrupts time-reversal symmetry [5]. In this scenario, the proximity of electric charges to the interface gives rise to a coexistence of dielectric and magnetoelectric polarization, which, in turn, triggers the generation of Hall currents and local magnetic fields [1]. Achieving local magnetization often involves the introduction of transition metal elements through chemical doping, with terahertz detection methods serving as the means to observe and quantify the resultant TME [4]. Various interface geometries, ranging from planar to spherical and cylindrical boundaries, have been meticulously modeled to explore and understand the intricate behavior of the TME [2].

A particularly promising avenue for advancing our understanding of the TME lies in the realm of semiconductor nanostructures. Their diminutive dimensions, existing within the quantum regime, offer a unique vantage point for the meticulous observation of nuanced effects [6]. Furthermore, the capability for controlled fabrication facilitates the creation of a diverse array of boundary geometries [7]. The curvature and symmetrical attributes of these interfaces provide a fertile ground for the investigation of induced electromagnetic fields, and it is within this context that our study embarks upon a systematic exploration of how the size and shape of nanostructures can exert influence on magnetic fields brought about by point charges situated in proximity to TI-OI interfaces. This investigation is carried out with a specific emphasis on the behavior within quantum wells embedded within TI materials.

2. Methods

We employ the method of image dyons, a highly promising approach for assessing magnetic field variations [8]. Our analysis focuses on a specific configuration, where a semiconductor QW is enveloped by a TI, featuring two parallel planar interfaces [9]. This unique setup enables us to manipulate the resulting magnetic field through interactions between these interfaces. It's worth noting that, in this paper, we refrain from an in-depth discussion of the mathematical formulations, opting instead to concentrate on presenting results for two distinct scenarios involving topological QWs with a charge denoted as Q. It's noteworthy that our findings have been validated using a versatile numerical model based on the finite element method to solve Maxwell's equations [1]. Furthermore, electric charges located near the interface of topological (TI) and ordinary (OI) insulators induce magnetic fields within the medium, a phenomenon well-described by the method of image dyons – an extension of the method of image charges in classical electrostatics. Within this context, we recurrently calculate the image dyons and the resulting magnetoelectric potentials in a system that comprises two planar-parallel OI-TI interfaces, forming a finite-width slab. This methodology allows us to model magnetoelectric fields in topological quantum wells, thin films, and layers of two-dimensional materials.

The modeled system and related parameters are sketched in Figure 1 for better understanding where an electrostatic charge q_0 is placed on top or inside a w-width slab with OI-TI interfaces. We assume that: The upper (U) interface is located at $z = 0$. The lower (L) interface is located at $z = -w$. q_0 is located along the z-axis. We have either a TI slab between two OI [i.e., $(\Lambda_U, \Lambda_C, \Lambda_L) = (0, \pi, 0)$], or an OI slab between two TI [i.e., $(\Lambda_U, \Lambda_C, \Lambda_L) = (\pi, 0, \pi)$]. In the Figure 1, - q_0: electrostatic point charge. - z_0: absolute value of the position of q_0 in the z axis. - w: width of the slab. - α: Fine structure constant. - Dielectric constants: $\eta_U(z > 0)$, $\eta_C(0 > z > -w)$, $\eta_L(-w > z)$ - Magnetic permeabilities: $\mu_U(z > 0)$, $\mu_C(0 > z > -w)$, $\mu_L(-w > z)$

3. Results

In Figure 2, we illustrate the magnetic field distribution around a charge Q positioned at the center of a semiconductor QW with TI-OI interfaces. Strikingly, the top and bottom interfaces of the QW generate magnetic fields of opposing signs but equal magnitudes, resulting in the suppression of the magnetic field around Q. This behavior arises from constructive and destructive interference between the magnetic fields induced by these two interfaces. The presence of a nodal plane in the field distribution further underscores its similarity to the behavior of two quasi-dipoles with opposite signs. These results emphasize the profound influence of nanostructure geometry on the induced magnetic field, highlighting the potential for precise engineering of magnetic field configurations using the TME.

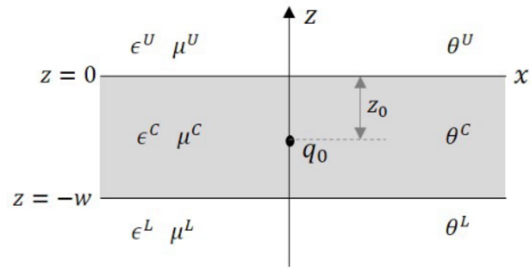

Figure 1. Schematic diagram of TME in a QW with a centered charge.

In Figure 3, we provide insight into the magnetic field distribution surrounding a charge Q situated 5 nm from the top interface of a semiconductor QW with TI-OI interfaces. Despite the reduced strength of the electric field reaching the bottom interface, the resulting magnetic field pattern closely mirrors that of a single plane. This similarity arises due to the diminished influence of the electric field on the bottom interface's magnetic field generation. However, subtle interplane interactions become apparent when examining the field distribution, suggesting that even in this configuration, the TME remains influential in shaping the resultant magnetic fields. These findings highlight the robustness of TME-driven magnetic field control and its applicability in varying nanostructure scenarios.

4. Discussion

4.1 Complex Magnetic Field Distribution

Figure 3 provides a more intricate picture of the magnetic field distribution compared to Figure 2. In the former, we analyzed the magnetic field around a charge placed off-center within the QW, which introduces a break in the system's symmetry. This departure from a symmetric setup leads to several notable observations:

1 **Strongest Near the Charge and Top Interface:** The magnetic field is most intense in proximity to the charge and at the top interface of the QW. This phenomenon results from the electric field generated by the charge. It is stronger at the top interface compared to the bottom interface, leading to a corresponding difference in magnetic field strength.

2 **Weakest at the Bottom Interface:** Conversely, the magnetic field is weakest at the bottom interface of the QW. This disparity in magnetic field strength is directly

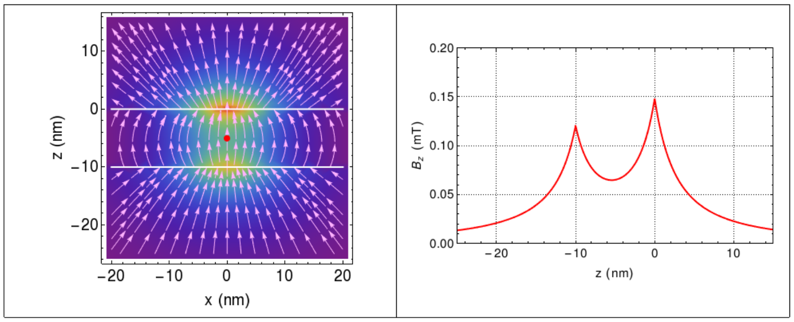

Figure 2. On the left panel, a TI-OI-TI configuration is shown mimicking TME in a QW with a centered charge. The dot at the center represents charge Q. On the right panel, a variation of magnetic field B is shown.

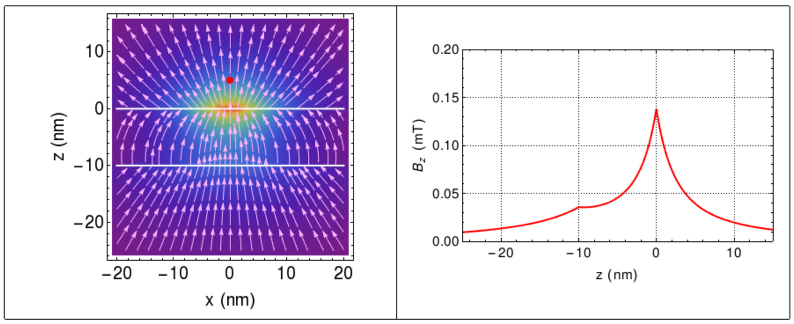

Figure 3. QW with an off-centered charge in a TI-OI-TI configuration is shown. The right panel shows variation of magnetic field B.

linked to the variation in electric field intensity across the QW.

3 **Perpendicular Magnetic Component:** Notably, the magnetic field exhibits a component perpendicular to the QW's plane. This unusual behavior is attributed to the TME. The TME uniquely couples electric and magnetic fields, meaning the electric field generated by the charge Q induces a magnetic field that is oriented perpendicular to the QW's plane. This perpendicular component is an intriguing feature that has implications for engineering magnetic field configurations.

4.2 Interplane Interactions and TME Influence

A critical aspect of our discussion centers on the subtle interplane interactions that become apparent when examining the magnetic field distribution in Figure 3. These interactions emphasize the continued influence of the topological magnetoelectric effect even in the off-centered charge configuration. Several key points come to the forefront:

1 **Electric Field Reaches the Bottom Interface:** Despite the charge's off-centered position, the electric field from the charge still reaches the bottom interface of the QW. However, it does so with a reduced intensity when compared to the top interface. This difference in electric field intensity is a crucial factor contributing to the interplane interactions.

2 **Opposite-Sign Magnetic Fields:** The electric field at the bottom interface generates a magnetic field, but this magnetic field has an opposite sign compared to the magnetic field induced by the top interface. The opposing signs result from the differing orientations of the electric field and magnetic field at the two interfaces.

3 **Dominance of Top Interface Magnetic Field:** Although the bottom interface contributes a magnetic field of the opposite sign, it is notably weaker than the magnetic field generated by the top interface. As a consequence, the magnetic field from the top interface predominates in the overall magnetic field distribution. This dominance emphasizes the robust and influential nature of the topological magnetoelectric effect even in asymmetric configurations.

5. Potential Applications

The applications of these findings hold great promise across multiple domains. They include the potential to revolutionize transistor design, allowing for current control via magnetic fields, resulting in more efficient and interference-resistant transistors. Additionally, the integration of the topological magnetoelectric effect into spintronic devices could lead to energy-efficient devices with larger storage capacities, enhancing their performance in information storage and processing applications. Moreover, advanced quantum sensors, which are vital for precise measurements of physical quantities, could be developed by harnessing the TME, offering heightened sensitivity and a broader operating range, thereby impacting fields such as magnetic field measurement and temperature sensing. These applications, driven by the control of magnetic fields, represent exciting opportunities for innovation and advancement in technology and materials science.

6. Conclusion

In conclusion, our study underscores the remarkable potential of the method of image dyons as a powerful and promising tool for analyzing the complex interactions of magnetic fields in nanostructures near TI-OI interfaces. The precise control of magnetic field configurations is revealed, offering a new avenue for innovation in materials science and technology. Importantly, the method of image dyons consistently provided accurate results, aligning closely with previous studies. This consistency underscores its reliability and robustness as an analytical technique, providing valuable insights into the behavior of magnetic fields in these intricate systems. These findings, in agreement with prior research, enhance our understanding of the topological magnetoelectric effect and its practical applications, shaping the future of condensed matter physics and technological advancements.

Acknowledgment

DJG acknowledges the contribution and support of the European Cooperation in Science and Technology (COST) Action CA21136 - “Addressing observational tensions in cosmology with systematics and fundamental physics (CosmoVerse).”

Data Availability and Source Code

Data and Python source code used in the study can be provided upon request for legitimate purposes, with proper citations/mentions.

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